Topic: Solving with conjugate method

Question: Find the limit.

$$\lim_{m \to 0} \frac{\sqrt{m+4}-2}{m}$$

Answer choices:

A 0

B 4

C $\frac{1}{4}$

D ∞

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Solution: C

The limit of

$$f(m) = \frac{\sqrt{m+4}-2}{m}$$

as m approaches 0 can't be found by direct substitution because we would get a zero in the denominator when $m = 0$. However, since we actually get $0/0$ by direct substitution, this is a clue that the zero will divide out of the denominator if we rewrite the function.

Since we have a radical involved in the function, we rationalize the numerator in order to try to divide out the zero in the denominator.

$$f(m) = \frac{\sqrt{m+4}-2}{m} \cdot \frac{\sqrt{m+4}+2}{\sqrt{m+4}+2}$$

$$f(m) = \frac{m+4-4}{m\left(\sqrt{m+4}+2\right)}$$

$$f(m) = \frac{m}{m\left(\sqrt{m+4}+2\right)}$$

$$f(m) = \frac{1}{\sqrt{m+4}+2}$$

Now that we've rewritten the function using conjugate method, we'll try the limit again using substitution.

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$$\lim_{m\to 0} f(m) = \lim_{m\to 0} \frac{1}{\sqrt{m+4}+2}$$

$$\lim_{m\to 0} \frac{1}{\sqrt{0+4}+2}$$

$$\frac{1}{2+2}$$

$$\frac{1}{4}$$

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Topic: Solving with conjugate method

Question: Evaluate the limit.

$$\lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3}$$

Answer choices:

A 6

B 3

C 9

D 0

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Solution: A

To solve a limit by conjugate method, multiply the numerator and the denominator by the conjugate of the denominator. This looks like

$$\lim_{x\to a}\frac{a+b}{c+d}=\frac{a+b}{c+d}\times\frac{c-d}{c-d}$$

or

$$\lim_{x\to a}\frac{a+b}{c-d}=\frac{a+b}{c-d}\times\frac{c+d}{c+d}$$

To solve this limit, we'll try direct substitution.

$$\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}$$

$$\frac{9-9}{\sqrt{9}-3}$$

$$\frac{9-9}{3-3}$$

$$\frac{0}{0}$$

Because we get an indeterminate form with substitution, and because we can't factor the numerator or denominator, we'll try conjugate method.

$$\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}\times\frac{\sqrt{x}+3}{\sqrt{x}+3}$$

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$$\lim_{x\to 9}\frac{(x-9)\left(\sqrt{x}+3\right)}{x-3\sqrt{x}+3\sqrt{x}-9}$$

$$\lim_{x\to 9}\frac{(x-9)\left(\sqrt{x}+3\right)}{x-9}$$

Cancel common factors.

$$\lim_{x\to 9}\sqrt{x}+3$$

Now we can try substituting again.

$$\sqrt{9}+3$$

$$3+3$$

$$6$$

This is the value of the limit.

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Topic: Solving with conjugate method

Question: Evaluate the limit.

$$\lim_{x \to 16} \frac{16 - x}{4 - \sqrt{x}}$$

Answer choices:

A 4

B 8

C 0

D 16

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Solution: B

To solve a limit by conjugate method, multiply the numerator and the denominator by the conjugate of the denominator. This looks like

$$\lim_{x\to a}\frac{a+b}{c+d}=\frac{a+b}{c+d}\times\frac{c-d}{c-d}$$

or

$$\lim_{x\to a}\frac{a+b}{c-d}=\frac{a+b}{c-d}\times\frac{c+d}{c+d}$$

To solve this limit, we'll try direct substitution.

$$\lim_{x\to 16}\frac{16-x}{4-\sqrt{x}}$$

$$\frac{16-16}{4-\sqrt{16}}$$

$$\frac{16-16}{4-4}$$

$$\frac{0}{0}$$

Because we get an indeterminate form with substitution, and because we can't factor the numerator or denominator, we'll try conjugate method.

$$\lim_{x\to 16}\frac{16-x}{4-\sqrt{x}}\times\frac{4+\sqrt{x}}{4+\sqrt{x}}$$

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$$\lim_{x\to 16} \frac{(16-x)\left(4+\sqrt{x}\right)}{16+4\sqrt{x}-4\sqrt{x}-x}$$

$$\lim_{x\to 16} \frac{(16-x)\left(4+\sqrt{x}\right)}{16-x}$$

Cancel common factors.

$$\lim_{x\to 16} 4+\sqrt{x}$$

Now we can try substituting again.

$$4+\sqrt{16}$$

$$4+4$$

$$8$$

This is the value of the limit.

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